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Stability Analysis of Optimized PMU Placement using Hybrid and Individual TLBO-PSO Techniques

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Abstract In power system to optimized PMUs is a critical task to ensure maximum network observability while minimizing installation costs. This study presents a comparative analysis of three optimization techniques: Teaching-Learning-Based Optimization (TLBO), Particle Swarm Optimization (PSO), and a hybrid TLBO-PSO approach, focusing on their efficiency in determining the best PMU placements. Individual methods, such as TLBO and PSO, are often limited by longer computation times and the requirement for a higher number of PMUs to achieve full observability. In contrast, the hybrid TLBO-PSO method demonstrates significant improvements, consistently delivering solutions with fewer PMUs, faster computation times, and higher placement accuracy. By evaluating performance of these techniques on IEEE 14bus, 30bus and 57 bus systems through simulations conducted over 100 iterations for each method in every test case. The results highlight the hybrid approach's superior efficiency compared to individual methods. Furthermore, comparisons with prior research confirm that the hybrid TLBO-PSO approach is a robust and reliable solution for minimizing PMU installations while ensuring complete system observability.

Keywords: Optimal PMU placement(OPP), Particle Swarm Optimization (PSO), Teaching-Learning Based Optimization (TLBO), Hybrid TLBO-PSO Approach, redundancy index, Stability analysis,

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1. Introduction

This Phasor Measurement Units (PMUs) are essential devices for modern power system monitoring, introduced in the 1980s to provide fast and intelligent communication within smart grids. PMUs measure voltage and current phasors , accurately time-stamped via Global Positioning System (GPS) technology, which allows synchronization of data from widely dispersed locations across the power system. Compared to traditional SCADA systems, PMUs offer higher accuracy due to their faster sampling rates [4]. This synchro-phasor technology enables real-time monitoring, fault detection, protection, and control, enhancing the overall reliability of the power grid [5]. Once data is collected by PMUs, it is transmitted through optical fiber to a centralized Phasor Data Concentrator (PDC), where the data is aligned and stored for post-dispatch analysis. This process supports stability by allowing timely actions in response to grid events. The challenge is to find optimized PMUs and their optimal locations to ensure complete observability of power system networks. Earlier various optimization techniques have been used to solved the optimization problem effectively.

1.1. Analysis of optimal PMUs placement methods

Conventional and heuristic optimization methods have been applied to the Optimal PMU Placement (OPP) problem. Conventional methods, such as Integer Linear Programming (ILP), used for optimal PMUs required for full observability .The Conventional Technique works well for simpler problems but requires more computation time, making it less effective for complex tasks. It is also less accurate because it is based on many physical assumptions. On the other hand, the Heuristic Technique is well-suited for solving complex problems by mimicking human-like decision-making, and it relies mainly on data with fewer assumptions. This leads to higher accuracy and faster performance, making it ideal for time-sensitive applications. Table 1 compares the properties of Conventional and Heuristic techniques in the context of PMU.

Table 1. Comparative analysis of Conventional and Heuristic Technique

S.No.	Properties	Conventional Techniques	Heuristic Techniques
1.	Size of Problem	Works well for simpler networks.	Solves complex problems by mimicking human-like methods.
2.	Data Requirement	Based on physical principles and assumptions.	Relies mainly on data with fewer assumptions.
3.	Accuracy	Less accurate due to multiple assumptions.	More precise with minimal failures.
4.	Effectiveness	Slower, requires more computation time.	Faster, ideal for time-intensive tasks.

These techniques highlight the need to balance computational efficiency, scalability, solution precision, and the ability to manage uncertainties within the power grid. By incorporating hybrid methods, the performance of individual algorithms is significantly improved, offering the advantages of multiple approaches and delivering enhanced optimization outcomes.

1.2 PMU placement formulation

The main objective is to find the minimum number of Phasor Measurment Unit required and its best location to monitor complete power system to achieve complete observability for the power network. Thus, the Objective function is formulated as below

$$minimize \sum_{k=1}^{N} x_k \tag{1}$$

Where N is a number of system buses and x_k is a binary decision variable, where:

$$x_k = \begin{cases} 1, & if\ pmu\ installed\ at\ bus\ k, \\ 0, & otherwise \end{cases}$$

Constraints:

The constraint ensures that every bus in the network is observable either directly by PMU at that perticular bus or indirectly through its connected neighbours.

$$[A]\times [X] \geq [b]$$

where [A] is a binary connectivity matrix of size NxN. Entries for matrix [A] are defined as follows:

$$A_{ij} = \begin{cases} 1 \; if \; i \; = \; j \\ 1 \; if \; i \; and \; j \; are \; connected \\ 0 \; if \; otherwise \end{cases}$$

Meanwhile [X] is defined as a binary decision variable vector of size Nx1 define as:

$$[X] = [x_1, x_2, x_3 \dots \dots \dots \dots . x_N]^T$$

[b] is a column vector of ones, of size Nx1, representing the requirement for each bus to be observable

$$[b] = [1,1,1 \dots \dots \dots \dots .1]^T$$

Fitness Function Formulation:

Given the components, the fitness function for the hybrid TLBO-PSO technique can be expressed as:

$$f(x) = \sum_{i=1}^{N} x_i + \lambda \times Penalty(x) \tag{2}$$

Penalty Function: It penalizes solutions that do not satisfy the observability constraint. If a solution does not meet the constraint, a penalty proportional to the violation is added to the fitness value. λ is a penalty factor (a large positive constant) that ensures the constraint violation has a significant impact on the fitness value.

The penalty function could be defined as:

$$Penalty(x) = \begin{cases} 0 \; , if \; [A]x[X] \geq [b] \\ Large \; positive \; value \; , if \; [A]x \, [X] \leq [b] \end{cases}$$

Final Fitness Function:

$$f(x) = Min[\sum_{i=1}^{N} x_i + \lambda \times max(0,[b] - [A]\times[X])] \tag{3}$$

The first term $\sum_{i=1}^{N} x_i$ encourages minimizing the number of PMUs.

The second term λ×max(0,[b]− [A]×[X]) adds a penalty if the solution violates the observability constraint.

SORI i.e. system observability redundancy index which is sum of bus observability for all buses. Maximizing the measurement redundancy value ensures that a major portion of the system remains observable in case of a PMU failure.

2. Research Methods

2.1Particle Swarm Optimization (PSO)

The fundamental principle of (PSO) (de Valle et al., 2008) is inspired by behavior of a flock of birds, a school of fish, or a swarm of bees. In PSO, multiple agents, or particles, are utilized to search for the optimal solution to the problem at hand. The movement of these particles toward the optimal solution is influenced by both their individual experiences and the collective knowledge of the swarm. As illustrated below, the position of a particle at any given moment is determined by its current velocity and its position from the previous moment

$$x_i^{(t+1)} = x_i^t + v_i^{(t+1)} \tag{4}$$

Where $x_i^{(t+1)}$ and x_i^t is a vector which indicates position of i_{th} particle at time instant t+1 and t respectively, and $v_i^{(t+1)}$ is the velocity vector of the particle.

The velocity vector is updated by using the experience of the individual particles, as well as the knowledge of the performance of the other particles in its neighbourhood. The velocity update rule for a basic PSO is

$$v_i^{(t+1)} = w * v_i^t + c_1 * r_1 * \left(p_{best_i} - x_i^t\right) + c_2 * r_2 * (g_{best} - x_i^t) \tag{5}$$

where $i = 1,2,3 \ldots\ldots\ldots, n$ and n is population size, v_i^t is the particle i velocity at time t, x_i^t is the position of particle i at time t,w is the inertia weight, which balances exploration and exploitation, c_1 and c_2 are cognitive and social coefficients, respectively, r_1 and r_2 are random values uniformly distributed in [0,1], p_{best_i} is the personal best position of particle i , g_{best}, is the global best position found by the swarm.

2.2Teaching Learning Based Optimization Algorithm (TLBO)

Teaching-Learning-Based Optimization (TLBO) is an optimization method inspired by the process of teaching and learning in a classroom. Proposed by Rao, Savsani, and Vakharia in 2011, TLBO mimics the impact of a teacher on learners and the interaction among learners to improve their knowledge. In the TLBO algorithm, the population consists of learners (solutions). The algorithm operates in two phases: the "Teacher Phase" and the "Learner Phase."

Teacher Phase: In this phase, the best result in the population, taken as as the teacher ($x_{Teacher}$), and always tries to betterment the mean result of the class (population) by moving the learners towards the teacher's knowledge level. The update rule in this phase is:

$$x_{new} = x_i + r * (x_{Teacher} - T_F . M) \qquad (6)$$

x_i is the current solution of i , $x_{Teacher}$ indicates best solution in the current population (teacher), T_F is the teaching factor, typically set to either 1 or 2, M is the mean of the current population, is r a random number of[0,1].

Learner Phase: In this phase, learners increase their knowledge by interacting with each other. Each learner pairs with another random learner and updates its solution based on the knowledge gained from the interaction. The update rule in this phase is:

$$x_{new,i} = x_{old,i} + r_i(x_j - x_i) \ \text{ if } (x_j) < (x_i) \qquad (7)$$

$$x_{new,i} = x_{old,i} + r_i(x_i - x_j) \ \text{ if } (x_i < (x_i) \qquad (8)$$

$x_i$representing the current solution in the solution space, x_j representing a different solution in the solution space, r is a scalar, typically a random number of [0,1] or a parameter that controls the step size of the update.

2.3 Hybrid TLBO-PSO Algorithm

The TLBO algorithm emulates the behavior of the teacher teaching the student and the student learning from the teacher. The PSO algorithm emulates the food-searching behavior algorithm for birds. The fitness function defined in figure 2 is subjected to convergence by using the hybrid algorithm of TLBO-PSO that helps in better convergence in the defined problem. The fitness function is design for optimal PMU placement in power system ensuring complete observsbility.

The hybrid algorithm combines the PSO update rules with the TLBO teacher and learner phases. TLBO is using as local search capacity. In PSO algorithm at each iteration, the particles' velocities and positions are updated, followed by improvements using TLBO's teacher and learner phases. The fundamental concept of hybrid TLBO-PSO is to combine the social thinking capacity (g_{best}) in PSO with TLBO's local search capacity. Finally g_{best} is considered as best optimal solution for PMU placement.The output of TLBO result given for initialization for PSO algorithm. The Hybrid algorithm implementation process is given by flow chart given below.

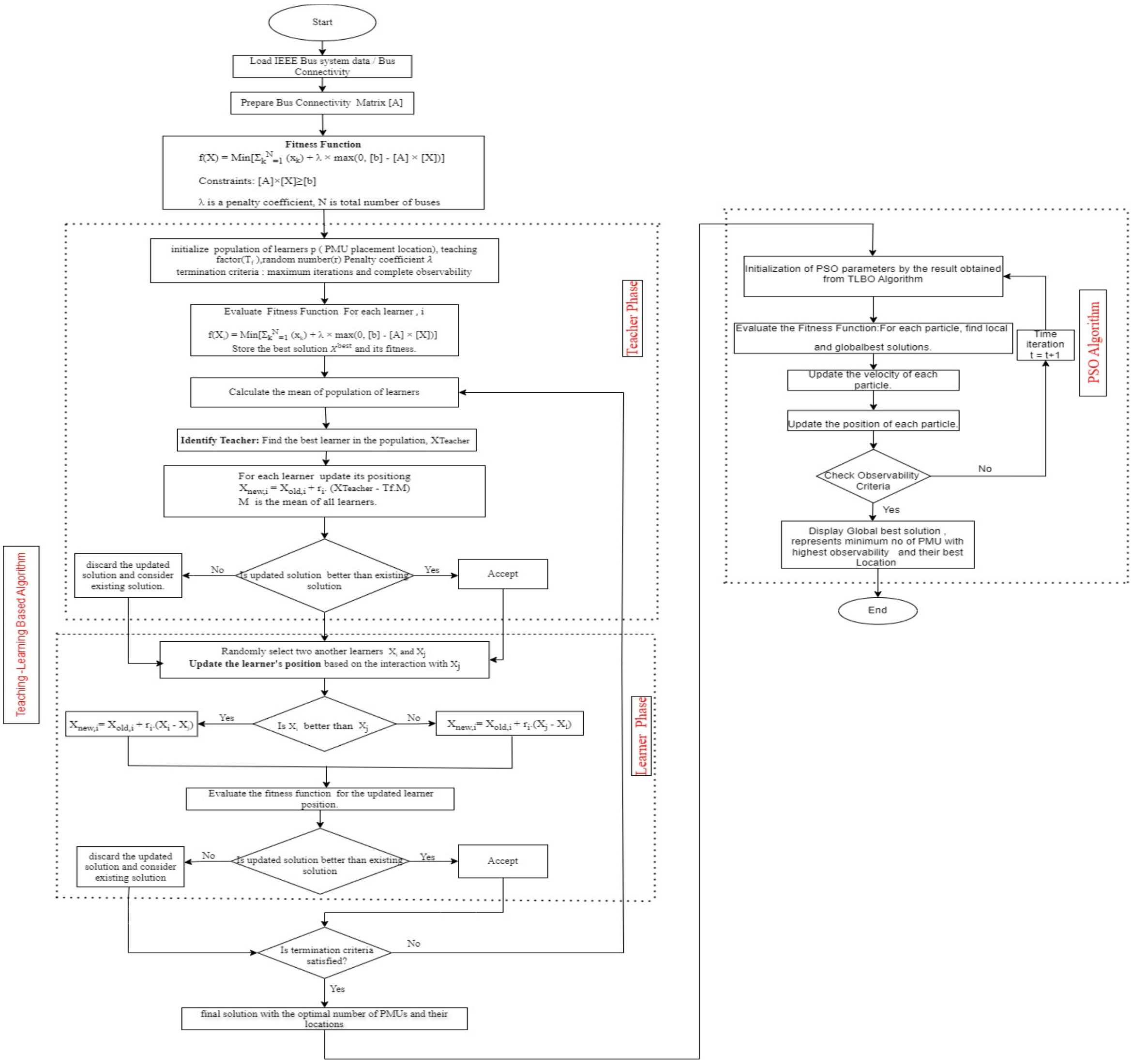

Figure 1. Flow chart for Hybrid TLBO -PSO method

3. Results and Discussion

Simulations is performed on IEEE 14, 30 and 57 bus test systems. The simulations are carried out in MATLAB 2021a environment on intel(R) Core(TM) i5-8265U CPU @ 1.60GHz 1.80 GHz, 64-bit operating system, x64-based processor. All three methods are executed for IEEE 14, 30 and 57bus systems for normal cases without considering ZIB and other factors and find the result for optimal placement of PMU's, various possible PMU numbers and their best locations and count of each PMU location for every 100 iteration. The results obtained are as follows for each systems: Simulations are performed on IEEE 14, 30 and 57 bus test systems.

The **Hybrid Method** consistently achieves the best solution with the minimum number of PMUs.

The **PSO** method shows less consistency than the Hybrid Method and the **TLBO** method tends to indicating slightly less optimal results compared to the Hybrid Method.

Case 1: IEEE 14 Bus System:

Fitness values (Min. no of PMU),Possible PMU Locations, SORI and No. buses observed multiple times once of PSO, TLBO, and proposed TLBO-PSO algorithms for IEEE 14 bus system are tabulated in Table 2.

Table 2. Different solutions for IEEE 14 bus system

Min no of PMUs	Location of PMUs	SORI	No. buses observed multiple times
4	2,6,7,9	19	4
4	2,7,11,13	16	2
4	2,6,8,9	17	3
4	2,7,10,13	16	2
4	2,8,10,13	14	0

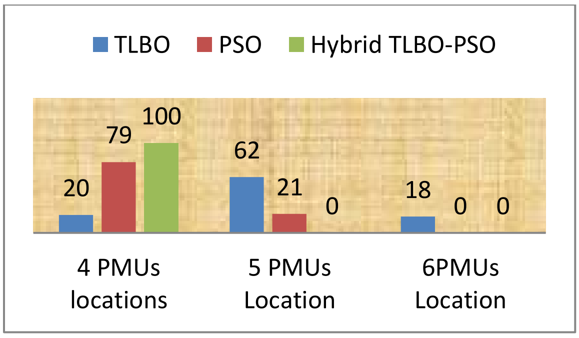

Figure 2. comparison of PMU Number for 100 iterations for IEEE 14 Bus system

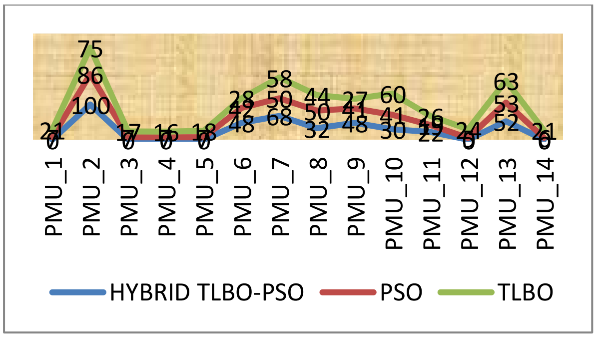

Figure 3. Comparative analysis of PMUs Count for 100 solutions for IEEE 14 Bus

Figure 2 clearly shows that the PMUs requirement are less for the TLBO-PSO algorithm than the PSO and TLBO algorithm. Figure 3 shows the result for of PMUs Count for 100 iteration. These underscores the effectiveness of the Hybrid Method in minimizing the number of PMUs, as it achieves the optimal count in all iterations.

Case 2: IEEE 30 Bus System:

The comparative results obtained from the all three methods are shown in table 3. The best SORI value achieved is 52, with a minimum of 10 PMUs necessary for complete observability and 12 buses observed more than once.

Table 3. Different Solutions for IEEE 30 Bus System

Min no of PMUs	Optimal Location of PMUs	SORI	No. buses observed multiple times once
10	2,4,6,9,10,12,15,20,25,27	52	12
10	2,4,6,9,10,12,15,18,25,27	52	12
10	1,2,6,9,10,12, 15, 19, 25, 27	50	14
10	2,4,6,9,10,12,19,24,25,27	51	12
10	1,2,6,9,10,12,15,20,25,29	48	11
10	2, 4, 6, 10, 11, 12, 19, 24, 30	46	10

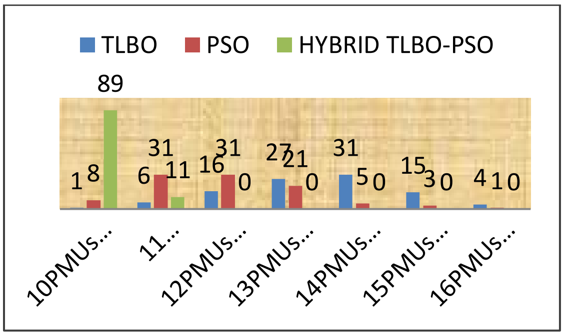

Figure 4. Comparison of PMU Number and location for 100 iterations for IEEE 30 Bus system

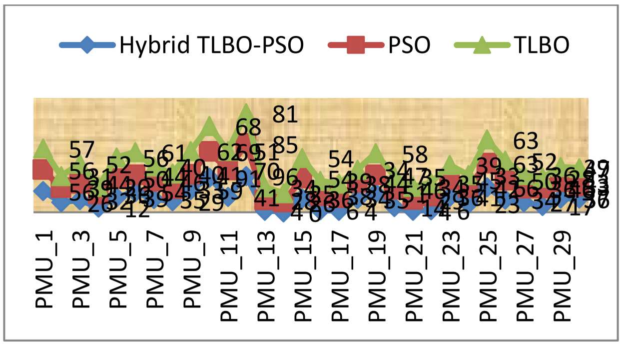

Figure 5. Comparative analysis of PMUs Count for 100 solutions for IEEE 30 Bus system

Figure 4 illustrates the superior performance of the hybrid algorithm compared to the other two methods. In 100 iterations, the hybrid algorithm achieved the minimum number of PMUs in 89 instances, whereas PSO and TLBO achieved this only 8 and 1 times, respectively. Additionally, the maximum number of PMUs required by the hybrid method is only 11, significantly lower than the other methods, which require up to 16 PMUs. Figure 5 Comparative analysis of PMUs Count for 100 solutions for IEEE 30 Bus system.

Case 3: IEEE 57 Bus System:

The comparative results obtained from the all three methods are shown in table4. The best SORI value achieved is 72, with a 17 PMUs gives for complete observability. Additionally, 15 buses observed more than once.

Table 4. Different Solutions For IEEE 57 Bus System

Min no of PMUs	Locations of PMUs (P)	SORI	No. buses observed multiple times
17	1,4,6,9,15,20,24,25,28,32,36,38,41,46,51,53,57	72	15
17	1,4,6,9,15,20,24,25,28,32,36,38,41,47,51,53,57	72	15
17	1,4,9,15,20,22,25,26,29,32,36,38,41,46,50,53,57	71	13
17	1,4,9,15,20,24,28,29,31,32,36,38,41,47,50,54,57	71	14
17	1,6,13,15,19,22,25,27,32,36,38,41,47,51,52,55,57	70	12

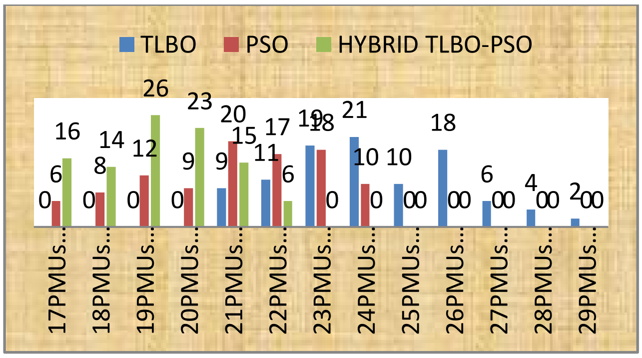

Figure 6. Comparison of PMU Number and location for 100 iterations for IEEE 57 Bus system

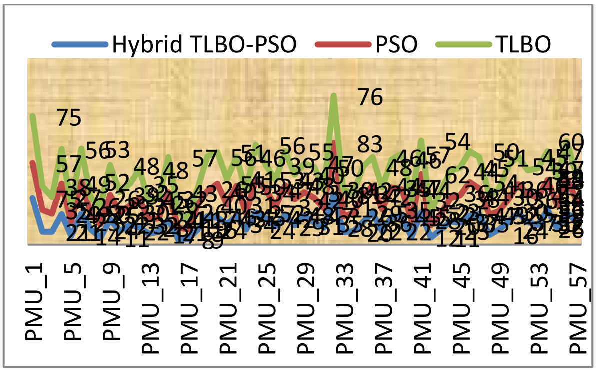

Figure 7. Comparative analysis of PMUs Count for 100 solutions for IEEE 57 Bus

Figure 7 illustrates the superior performance of the hybrid algorithm compared to the other two methods. In 100 iterations, the hybrid algorithm achieved the minimum number of PMUs in 16 instances, whereas PSO and TLBO achieved this only 6 and 0 times, respectively. Additionally, the maximum number of PMUs required by the hybrid method is only 17, significantly lower than the other methods, which require up to 16 PMUs. Figure 6 Comparative analysis of PMUs Count for 100 solutions for IEEE 57 Bus system.

Table 5: The Hybrid TLBO-PSO method demonstrates superior computational efficiency. Average execution times are significantly faster compared to PSO,TLBO, Modified BPSO and ABC algorithms.

Table 5. Comparative analysis of Computation Efficiency for various IEEE systems

Method	Average execution time in (seconds)		
	IEEE 14 Bus	**IEEE 30 Bus**	**IEEE 57 Bus**
Hybrid TLBO-PSO	**3.9407**	**16.8493**	**34.3346**
PSO	17.2106	44.8387	55.7126
TLBO	45.4108	87.0084	145.1668
Artificial Bee Colony (ABC) Kulanthaisamy et al. (2014)	40	54	303
Modified BPSO, Hajian and Ranjbar	60	360	2580

Table 6: The Hybrid TLBO-PSO method exhibits the highest stability for all cases in comparison, PSO and TLBO show lower stability. This table highlighting the robustness of the hybrid approach in achieving consistent optimal solutions.

Table 6. Comparative analysis of stability of each method

Method	Analysis of achieving the optimal solutions for 100 trials (Stability)		
	14 bus	**30 bus**	**57 bus**
Hybrid TLBO-PSO	100	89	16
PSO	79	24	6
TLBO	20	7	1

Table 7**:** The percentage of PMUs required for observability is as follows: 28% for 14 Bus**,** 33% for 30 Bus**,** 29% for 57 Bus, and 27% for 118 Bus**,** demonstrating efficient PMU placement relative to system size.

Table 7. Optimal Solutions for various IEEE systems

IEEE Bus Systems	Min. no. PMUs	Locations of PMUs	n PMU /N, %	SORI
14 bus	4	2, 6, 7, 9	28%	19
30 bus	10	1,2,6,9,10,12, 15, 19, 25, 27	33%	50
57 bus	17	1,4,6,9,15,20,24,25,28,32,36,38,41,46,51,53,57	29%	72

4. Conclusion

After conducting 100 trial runs, the optimized PMUs and their locations are determined. All the algorithms tested successfully provided best solutions for the PMU placement problem. However, the TLBO algorithm, while effective, required more computational time and did not consistently yield superior results. In contrast, the PSO algorithm performed better, offering improved results with shorter computation times. Most notably, the hybrid TLBO-PSO algorithm demonstrated superior reliability and accuracy, consistently achieving the minimum number of PMUs compared to the individual TLBO and PSO algorithms.

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