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Comparative Performance of GLMM and GEE for Longitudinal Beta Regression in Economic Inequality Modelling

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Abstract. Due to the shortcomings of conventional Gaussian methods, specialized models are frequently needed for longitudinal data analysis with bounded outcomes, such as the Gini ratio. In order to model economic inequality in Indonesia, this study compares the effectiveness of Generalized Linear Mixed Models (GLMM) and Generalized Estimating Equations (GEE) for beta-distributed longitudinal data. Root Mean Square Error (RMSE) and pseudo R-squared values are used to assess model performance using panel data from 10 provinces between 2018 and 2024 as well as important socioeconomic indicators. With lower RMSE and higher explanatory power across all provincial subsets, the results consistently demonstrate that GLMM performs better than both GEE and generalized linear models (GLM). ANOVA tests verify that modeling methodologies, not data heterogeneity in GRDP or Gini values, are responsible for the differences in model performance. These results demonstrate how well GLMM handles complex data structures and within-subject correlations, providing more accurate and effective estimates in longitudinal beta regression scenarios. The study encourages the use of GLMM for more precise longitudinal analysis in economic and social research and offers insightful information for researchers modeling inequality indices.

Keywords: Beta, GEE, GLMM, Gini Index, Economic Inequality

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1. Introduction

Longitudinal or panel data analysis has become increasingly vital across disciplines, including economics, health, and social sciences, due to its ability to capture temporal patterns and subject-level dynamics[1]. In many economic studies, researchers encounter response variables constrained within a bounded interval (0,1), such as proportions, indices, or ratios. One notable example is the Gini ratio, a widely used measure of economic inequality, which is always bounded between 0 and 1 and thus suitable for modeling using the beta distribution.

The Gini ratio and the factors that influence it are important topics in the study of development economics and public policy[2]. Standard linear regression models, particularly those based on the assumption of normality, are generally inappropriate for such data [3], [4], [5]. They may produce predictions outside the feasible interval, suffer from heteroscedasticity, and offer limited interpretability for proportional data [6]. This has led to the increasing use of beta regression models, especially following the influential formulation by Ferrari and Cribari-Neto [7], which allow for modeling responses within the (0,1) interval more effectively than Gaussian models. Prior findings also show that beta regression typically yields lower AIC and BIC values when compared with normal-based models in modeling ratio data [8].

In the context of longitudinal data, two widely adopted modeling frameworks are Generalized Linear Mixed Models (GLMM) and Generalized Estimating Equations (GEE) [9]. GLMM provides subject-specific inference by incorporating random effects, while GEE estimates population-averaged effects and is more robust to correlation structure misspecification. On the other hand, GEE is an approach that focuses on estimating the parameters of the average population, considering the correlation structure in longitudinal data. GEE is known for its robustness in the misspecification of correlation structures and its ability to produce consistent parameter estimates even when the correlation structures are not correctly specified.

However, despite their extensive application to count or binary data, there remains a lack of direct comparison between GLMM and GEE in the context of beta-distributed longitudinal responses, especially for modeling economic indices like the Gini ratio. Existing comparative studies on GLMM and GEE largely focus on Poisson or binomial models [10], [11], and their conclusions cannot be directly extended to continuous bounded outcomes.

An identifiable gap from previous studies is the lack of direct comparison between GLMM and GEE for longitudinal data with beta-distributed responses. There are limited studies that discuss the performance of both methods in the context of modeling economic indices such as the Gini ratio. There is a lack of exploration of how differences in data characteristics (e.g., sample size, number of observations per subject, level of intra-subject correlation) affect the relative performance of GLMM and GEE for beta-distributed data.

Given the importance of beta-distributed longitudinal data modeling and gaps in the existing literature, further research is needed to compare the performance of GLMM and GEE in this context. Such studies will significantly contribute to the development of statistical methodologies for longitudinal data analysis and provide practical guidance for researchers in selecting the most appropriate method for their data analysis.

This study addresses this gap by systematically comparing the performance of GLMM and GEE for beta-distributed longitudinal data, using the Gini ratio as a case in point. We employ provincial panel data from Indonesia spanning 2018 to 2024 to model economic inequality and its key determinants. The Gini ratio, which measures the inequality of income distribution, is always in the interval (0.1) and is therefore suitable for modeling using beta distributions. Factors affecting the Gini ratio include economic growth, poverty rate, human development index, and open unemployment rate. Longitudinal analysis can help understand how these factors affect economic inequality over time and how their effects vary between provinces.

This study aims to fill the gap by comparing the performance of GLMM and GEE in analyzing longitudinal data with beta-distributed responses, with a special focus on Gini ratio modeling. The results of this study are expected to provide better guidance for researchers in choosing the most suitable method for their data analysis and provide new insights into the dynamics of economic inequality in Indonesia. This paper contributes to the literature by: (1) offering the first empirical comparison of GLMM and GEE in the context of beta-distributed longitudinal data for inequality indices; and (2) providing practical guidance for applied researchers in selecting appropriate modeling frameworks for bounded panel data. While focused on Indonesia, the study’s findings have broader implications.

2. Methods

This study utilizes longitudinal (panel) data from 10 Indonesian provinces over a six-year period (2018–2024), sourced from official publications by BPS-Statistics Indonesia. The sample was selected to reflect diverse combinations of Gini ratio levels and GRDP per capita, as outlined in Table 1.

Table 1. Sample Province

Gini Ratio	GRDP	Province Sample
Low	High	North Sumatra and West Sumatra
Middle	Middle	Bali, West Kalimantan
Low	Low	Maluku, Bangka Belitung
High	High	West Java, East Java
High	Low	South Sulawesi, Southeast Sulawesi

The dependent variable is the Gini ratio, ranging from 0 to 1. Explanatory variables include:

Table 2. Research variables

Variable	Unit
Human Development Index	Points
Percentage of Poor People	Percent
Economic Growth	Percent
Open Unemployment Rate	Percent

The basic model of this study is the beta regression model. A beta regression model is used if the data follows a beta spread (the inlay is between 0 and 1)[12]. The beta distribution function can be written as follows [13]:

$$f(y;a,b)=\frac{\Gamma(a+b)}{\Gamma(b)\Gamma(b)}y^{a-1}(1-y)^{b-1} \qquad (1)$$

with 0 <y < 1; a > 0, b > 0, and Γ(.) are gamma functions.

while the Logistic Beta Regression model can be written with the equation:

$$g(\mu)=logit(\mu)=\ln\left[\frac{\mu}{1-\mu}\right]=¿\beta_0+\beta_1 x_1+\cdots+\beta_p x_p ¿ \qquad (2¿$$

with $\mu=\frac{e^{(X^T\beta)}}{1+e^{(X^T\beta)}}$[3]

The GLMM model equation in the case of the beta regression model is as follows:

$$g(\mu_{ij})=logit(\mu_{ij})=\log\left[\frac{\mu_{ij}}{1-\mu_{ij}}\right]=x_{ij}^T\beta+z_{ij}^T b_i dengan\, b_i \;\; N(0,G) \qquad (3)$$

In this case, it is a random covariate vector and shows a positive definitive covariance matrix of random effects, whereas generally, it is a scalar number (for random intercept models only) or a bivariate vector. $z_{ij}^T G\, b_i$.

The GEE equation in the case of the beta regression model is as follows:

$$g(\mu_{ij}) = logit(\mu_{ij}) = \log\left[\frac{\mu_{ij}}{1-\mu_{ij}}\right] = x_{ij}^T \beta \, dengan \, Var(Y_{ij} \vee x_{ij})_i = \phi \mu_{ij}(1-\mu_{ij}) \quad (4)$$

$$\sum_{i=1}^{N} D_i^T V_i^{-1}(Y_i - \mu_i) = 0 \text{ with } D_i = D_i(\beta) = \frac{\partial \mu_i(\beta)}{\partial \beta^T} \text{ [15]}$$

$D_i$is the diagonal matrix of the first derivative and = diag() is the diagonal matrix of the variance$V_i v_i t$

Criteria for choosing the best model

This study's model selection is based on RMSE (instead of using MSE as in [16]) and pseudo r square. The best-performing model is defined as the one with the lowest RMSE and highest pseudo R^2 across provinces [13], [17].

$$RMSE = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{N-P}} \quad (5)$$

Where:

$y_i$is the actual value of the ith observation.

$\hat{y}_i$ is the predicted value for the ith observation.

P is the number of the parameter estimated, including the constant.

N is the number of observations

Because beta regression does not rely on the assumption of normality, authors did not conduct tests for normality or homoscedasticity. The GLMM approach captures heterogeneity through random effects, while the GEE model takes into account autocorrelation within subjects. These features are not typically addressed by standard GLMs, making both models well-suited for handling non-normal, bounded longitudinal data.

A flowchart summarizing the model implementation steps, including data preparation, estimation, and comparison across GLM, GLMM, and GEE, is presented in Figure 1.

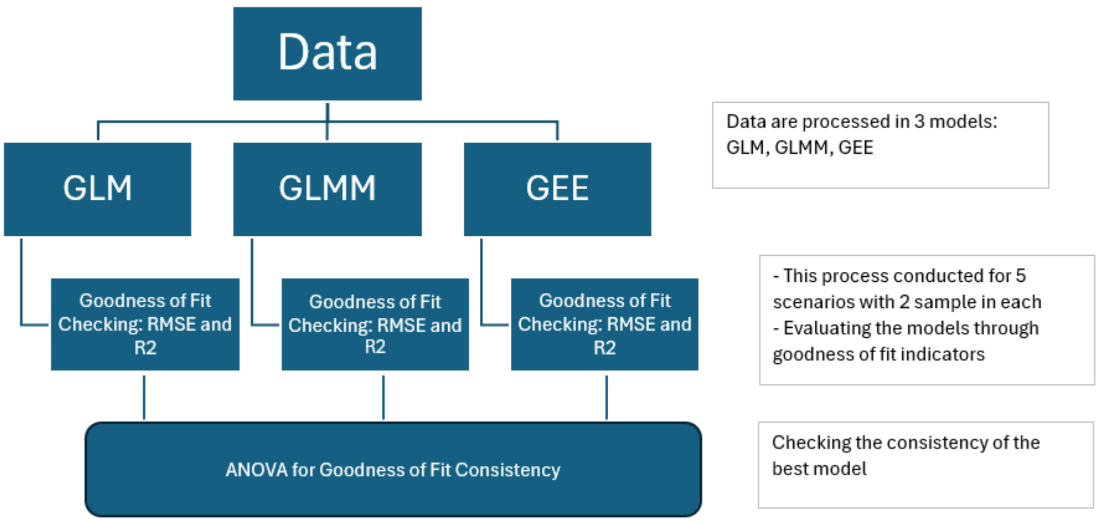

Figure 1. Workflow of beta regression modeling and model comparison using GLM, GLMM, and GEE approaches

3. Results and Discussion

This section presents the results of model comparisons using RMSE and pseudo R^2 values, followed by a detailed discussion of their implications for regional inequality analysis. This part also explain the relevance of these results in broader economic and policy contexts, connect the findings to previous studies, and acknowledge the study's limitations.

3.1. Quantitative Comparison of Model Performance

To evaluate the modeling performance, three models (Beta GLM, Beta GLMM, and Beta GEE) were applied to longitudinal Gini ratio data for 10 Indonesian provinces between 2018 and 2024. Each province represents a unique economic scenario, based on combinations of GRDP and Gini ratio levels. The results of each model's performance, as measured by RMSE and pseudo R^2, are presented in Figure 2.

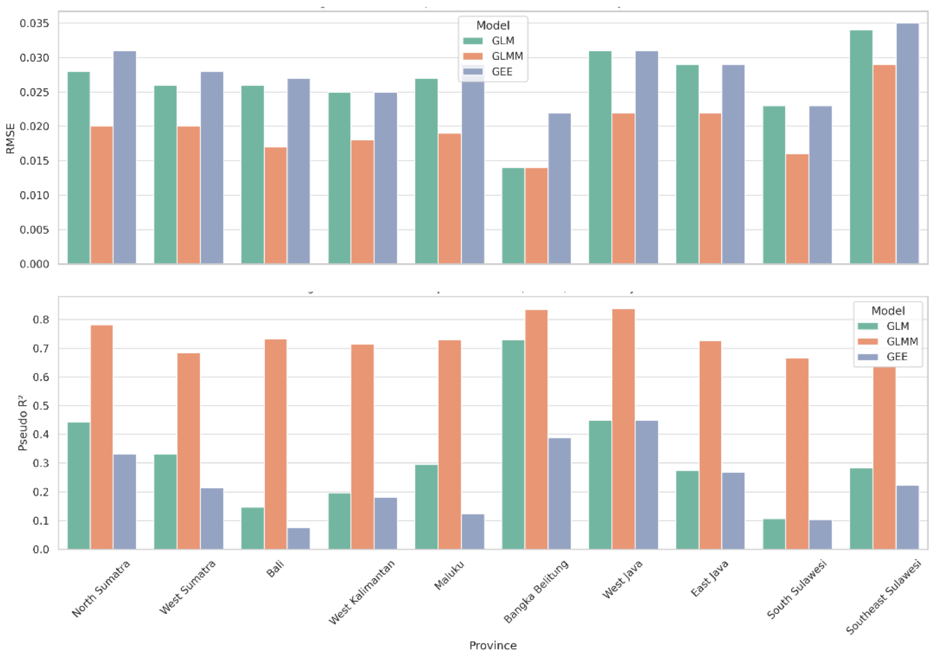

Figure 2. RMSE and Pseudo R2 for GLM, GLMM, and GEE

Across all provinces, GLMM clearly provides the best fit. It achieves an average RMSE improvement of over 30% compared to GLM and GEE, and consistently higher R^2 values. These results are especially striking in provinces with more volatile inequality trajectories, such as South Sulawesi, Bali, and East Java, where GLMM better captures dynamic local variation.

GLMM's superior performance is attributed to its use of random effects, which account for unobserved, time-invariant differences across provinces. This allows the model to "learn" province-specific inequality dynamics that would otherwise be flattened in population-averaged models like GEE.

Although GEE can still accommodate longitudinal structure and correlation, its robustness under correlation misspecification trades off with local sensitivity. GLM, while straightforward to implement, lacks both subject-specific modeling and support for intra-subject correlation, leading to underfitting and misinterpretation of regional trends.

The quantitative differences between models have practical implications. Lower RMSE values in GLMM indicate greater precision in estimating inequality at the provincial level. For policymakers, this means GLMM can support more accurate identification of regions in need of intervention, whether for social protection, fiscal support, or capacity development due to more accurate targeting of interventions [18], [19], [20].

In countries like Indonesia, which are characterized by economic and institutional decentralization as well as persistent regional inequality, such precision is essential. The trickle-down effect often fails to reach disadvantaged provinces, and aggregate national trends conceal significant local disparities [21], [22], [23]. GLMM's subject-level flexibility can guide more targeted and responsive policy planning.

For example, in provinces like West Java and Bangka Belitung, where GLMM significantly improves prediction accuracy, the model can inform customized inequality-reduction programs. Likewise, in lagging regions such as South Sulawesi, GLMM's stronger fit allows for better monitoring and forecasting of inequality trends, helping local governments align interventions with real-world trajectories.

3.2. Statistical Validation and Model Effects

To further test the robustness of the model differences, we conducted an ANOVA analysis on RMSE and R^2 values, using GRDP and Gini classification as covariates. The results are shown below.

Table 3. ANOVA Table

ANOVA - RMSE

Cases	Sum of Squares	df	Mean Square	F	p
Model 2	0.008	3	0.003	68.947	< .001
PDRB	5.246×10^{-5}	2	2.623×10^{-5}	0.661	0.526
Gini	7.500×10^{-7}	2	3.750×10^{-7}	0.009	0.991
Model 2 ✻ PDRB	3.342×10^{-5}	3	1.114×10^{-5}	0.281	0.839
Model 2 ✻ Gini	5.525×10^{-5}	3	1.842×10^{-5}	0.464	0.710
PDRB ✻ Gini		0			
Residuals	9.130×10^{-4}	23	3.970×10^{-5}		

Note. Type II Sum of Squares

ANOVA - R2

Cases	Sum of Squares	df	Mean Square	F	P
Model 2	1.699	3	0.566	31.642	< .001
PDRB	0.050	2	0.025	1.397	0.268
Gini	0.013	2	0.006	0.353	0.706
Model 2 ✻ PDRB	0.002	3	5.399×10^{-4}	0.030	0.993
Model 2 ✻ Gini	0.010	3	0.003	0.184	0.907
PDRB ✻ Gini		0			
Residuals	0.412	23	0.018		

Note. Type II Sum of Squares

From the above results, it can be seen that GLMM modeling has the largest R2 value compared to GLM and GEE modeling. On the other hand, GLMM has the smallest RMSE value compared to the other two models. This result is attenuated; GLMM extends the GLM model and incorporates random effects to account for within-subject correlation, making it suitable for subject-specific inferences. It can manage complicated data structures, such as non-normal distributions, and is based on maximum likelihood estimation [9], [24], [25]. Thus, GLMM can provide more accurate and efficient parameter estimation, especially when the correlation between observations is significant enough. GLMM model is better suited for data with complex hierarchical structures and can handle missing data more effectively [26], [27]. GEE is more robust in misspecifying the correlation structure and is more straightforward to implement for large samples.

Furthermore, the results are consistent with prior findings that highlight GLMM's advantages in handling nested, longitudinal data [14], [15]. Although past comparative studies often focused on Poisson or binomial outcomes in epidemiology and biostatistics, this study extends the analysis to bounded, continuous economic data using beta regression. Our findings reinforce this evidence while demonstrating GLMM's policy-relevant value in economic contexts. Few studies have directly compared GLMM and GEE for beta-distributed inequality data, making this contribution both novel and actionable for economic researchers and planners.

4. Conclusion

The study analyzed the performance of three statistical models: GLM, GLMM, and GEE, in the context of regional economic inequality using longitudinal data with beta distribution. Focusing on the Gini ratio over the ten Indonesian provinces from 2018 to 2024, the findings revealed that GLMM was the best model. It consistently yielded lower RMSE values and higher pseudo-R-squared scores, signifying better model fit and more explanatory power in terms of time and regional shifts relative to the other models.

The findings reported are effective not only from an academic angle but also from a pragmatic perspective for policy developers. It is worth mentioning that GLMM performs well in capturing within-province variation over time due to its random effects. In contrast, GEE concentrates on estimating average outcomes from all provinces, which could potentially be detrimental in situations where insights into localized contexts are needed. Moreover, although GLM is the most straightforward to achieve, it does not have the means to tackle the bounded and repeated data structure, which can potentially limit the scope.

This study has several limitations. First, it does not yet explore hybrid approaches that combine the local precision of GLMM with the robustness of GEE, which could improve model flexibility in complex settings. Second, the models assume linear relationships between predictors and the Gini ratio. Incorporating nonlinear structures, such as beta additive models or tree-based GLMMs, could yield deeper insights. Third, the analysis is limited to provincial-level data. While the sampled provinces represent diverse scenarios, more granular data (e.g., municipality-level) could reveal inequality patterns within provinces, which are crucial for local development planning. Lastly, the study does not include cross-validation or sensitivity analysis, as the primary goal was comparative evaluation. Future work could assess model generalizability using out-of-sample forecasting, bootstrapping, or time-series decomposition to improve prediction robustness.

Having the correct model is ideal for policymakers focusing on a region or those involved in developing the region. Particularly when studying inequality in a setting where the regional disparity is high, regional-specific dynamics need to be modelled alongside policies that can then use the advanced insights offered by the model.

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